FRACTURE ROUGHNESS IN THE ANNEALED FUSE MODEL

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ABSTRACT. We have studied the properties of fracture in an annealed fuse model, with focus on fracture roughness. The results are obtained via numerical simulation. We have found that neither H nor ν are universal quantities for the model. Both increase for increasing η. The results indicate an asymptotic value close to 1 for H as $\eta \to \infty$.

Fracture processes are interesting from a fundamental point of view, as well as being relevant to fields as diverse as material engineering and seismology. Especially intriguing from a physicist's perspective is the scaling properties for fracture surfaces, which has been shown to be universal for a wide range of materials [1, 2, 3, 4, 5, 6, 7].

The propagation of a crack in a material is obviously influenced by the strain distribution in the material. In addition, disorder in the material introduces a stochastic element to the fracture propagation. Together, these two properties decide the shape and roughness of the fracture surface.[8]

One can gain qualitative insights in the fracture process by studying simplified models, where complicating, non-necessary factors are stripped away. By capturing a few essential properties of the system, such as strain and disorder, important effects can be isolated and better understood. Several models should be studied, to make the final picture as complete as possible.

One particularly successful class of models for fracture is the fuse models [8, 9, 10], which simplify the description of a continuous material by instead considering a lattice of bonds. Each bond is represented as a linear resistor which can break if subjected to too high current. There are two different classes of fuse models, depending on how they treat disorder: the 'quenched' disorder models have a disorder built into the system, while the 'annealed' fuse models use a probabilistic breaking rule.

In this work we have looked at a fuse model with 'annealed' disorder, proposed by Hansen, Roux and Hinrichsen in 1990 [11]. The probabilistic breaking algorithm for the method is as follows:
1) to each bond assign a probability $p_i(t) = \frac{V_i(t)^\eta}{\sum_j V_j(t)^\eta}$ proportional with the strain V_i over the element, to the power of an exponent η ,

2) select one bond at random, but according to the probabilities $p_i(t)$ and
3) break this bond.

The model is similar to the Dielectric Breakdown model (DB) [12], except that the connection-requirement is dropped. (In DB, only neighbour bonds to the already existing cracks are candidates to break.) The annealed fuse model has been described in terms of its fractal dimensions by Hansen et al [11] and Curtin et al [13]. Fractal dimensions for DB is also known [14, 15]. However, the annealed fuse model has not earlier been studied with a focus on fracture roughness.

The results presented has been produced by numerical simulation of a two-dimensional, square lattice of fuses which is subjected to an externally applied stress. The stress is represented by an applied voltage over the lattice, with the principal axes of the lattice at $\pm 45°$ on the applied voltage. The current distribution is found by solving the Kirchoff equations using a conjugate gradient method [16].

The behaviour of the annealed fuse model at the limits $\eta \to 0^+$and $\eta \to +\infty$ can be described respectively by the models screened percolation[17] and Laplacian random walk (LRW)[18]. In the limit $\eta \to 0^+$, the selection process is random, but with the restriction that that only bonds within the current-carrying backbone can be broken. This restriction distinguishes screened percolation ($\eta \to 0^+$) from classical percolation ($\eta = 0$). When η becomes larger, the voltage distribution over the network plays a more and more important role. In the limit $\eta \to +\infty$ the element under the largest stress is always selected, and the model becomes equivalent to LRW. This corresponds to the growth of a single crack, since the highest stress always occurs at the tip of the crack [13].

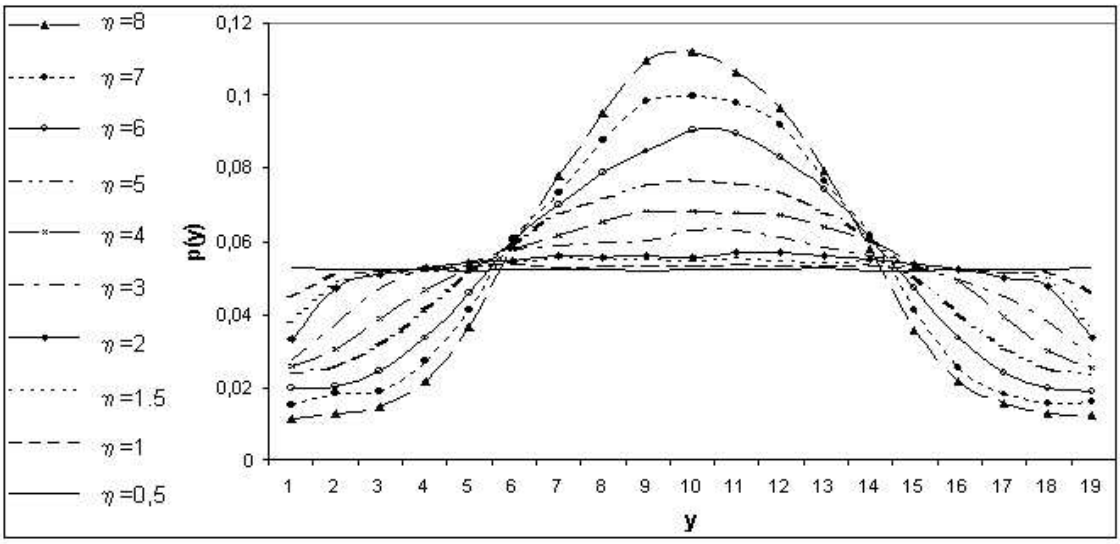


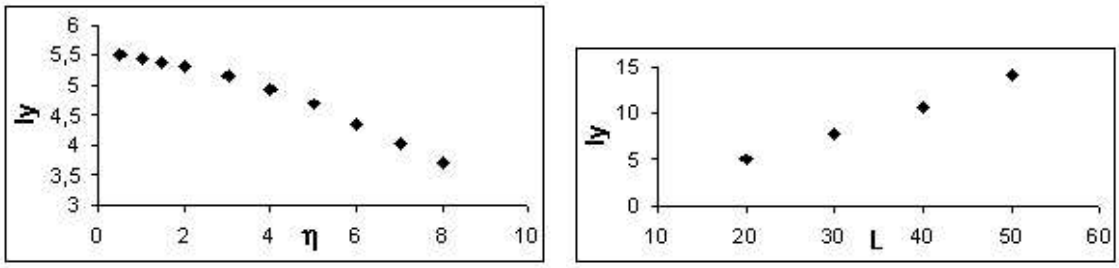
FIGURE 1. Damage profiles for $\eta = 0.5, 1, 1.5, 2, 3, 4, 5, 6, 7$ and 8.

A phase transition has been identified at $\eta = 2$ by Hansen et al [11]. For $\eta < 2$, failure develops trough proliferation of small defects, while at $\eta > 2$ the dominating

effect is the growth of one large cluster of broken bonds. For $\eta > 2$, the model therefore behaves like DB.

Figure 1 shows the damage profiles for various η. At very small η, the damage is virtually uniform over the sample, while at increasing η, it becomes more and more localised. It is apparent from the plot that there is a transition at approximately η between 2 and 3; for $\eta > 3$, the distribution has a clearly bell-shaped form.

The length scale characterising the width of the damage zone, $l_y = \frac{\sum(y-<y>)^2}{N-1}$, varies with η and L as shown in figure 2.



(a) (b)

FIGURE 2. (a) $l_y(\eta)$, (b) $l_y(L)$

For $\eta < 2$ we expect the model to behave in a percolation-like manner, with a diverging correlation length $\xi \propto |p - p_\infty|^{-\nu}$ [19], where p is the density of broken bonds at failure, while p_∞ is the density at which an infinite lattice breaks down. For classical percolation on a quadratic lattice in 2d, $\nu = \frac{4}{3}$ [20] . The fluctuations $(< p^2 > - < p >^2)^{1/2}$are expected to scale as $L^{-1/\nu}$ [21]. The result for $\eta = 0,5$, $\eta = 1$ and $\eta = 1,5$ is shown in figure 3. From this we find that ν increases with increasing η , as shown in figure 4.

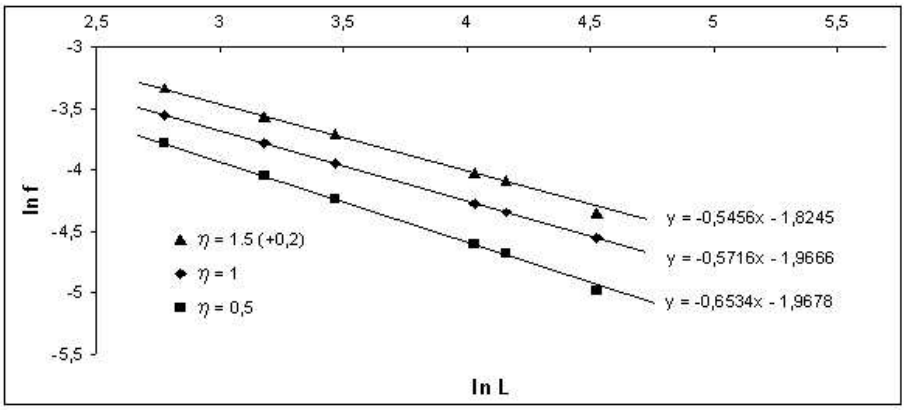


FIGURE 3. The fluctuations $f = (< p^2 > - < p >^2)^{1/2}$ as a function of L (ln-ln plot). The equations for the straight lines best fitted to the data are shown in the figure.

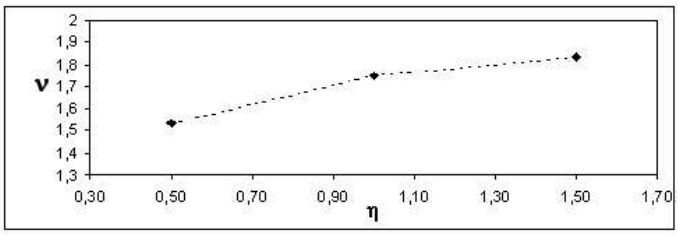
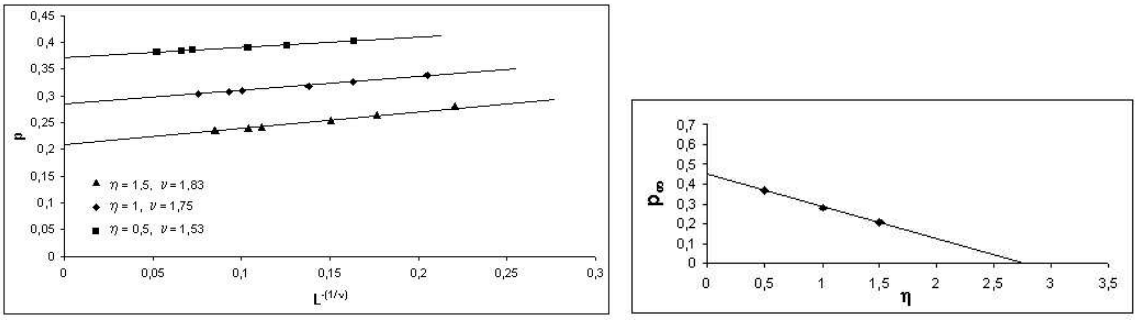


FIGURE 4. ν as a function of η.

We define the 50% survival probability, p_s as the density at which 50% of the samples survive. p_s then varies with L as $p_s = p_\infty - \frac{c}{L^{1/\nu}}$ [19] where c is a constant, as shown in figure 5 a). We have used this to find p_∞. Figure 5 b) shows p_∞ as a function of η.

For classical percolation in a 2d quadratic lattice, $p_\infty = \frac{1}{2}$ [20]. For our system, $p_\infty < \frac{1}{2}$ for all η, and we have $p_\infty \approx 0,45$ in the limit $\eta \to 0^+$. Screened percolation has a lower p_∞ than classical percolation [11]. Therefore, the result $p_\infty(\eta \to 0^+) < \frac{1}{2}$ is as expected.

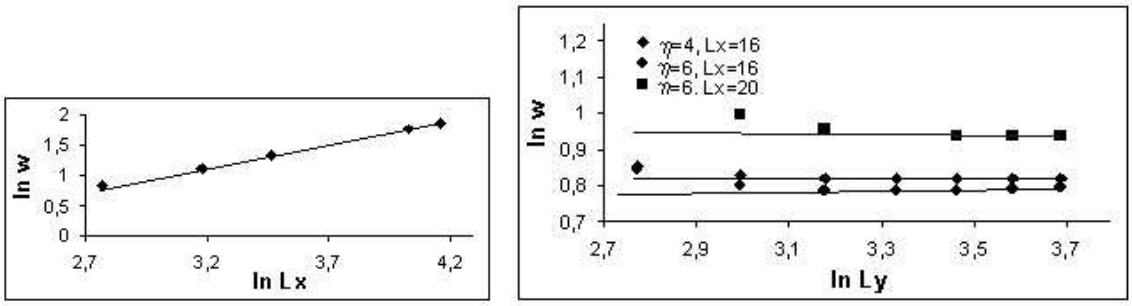


(a) (b)

FIGURE 5. (a) p_s as a function of $L^{-1/\nu}$ for various η (each with its corresponding ν). (b) p_∞ as a function of η.

As η increases from 0^+, p_∞ decreases. When the system enters the localisation regime, $p_\infty \to 0$. Our data give that $p_\infty(\eta)$ cross the η-axis for $\eta \approx 2,7$. This suggests that the transition from the percolation-like regime to the localisation-regime happens for η somewhat higher than expected from [11]; not at 2, but between 2 and 3. This is consistent with the damage profiles (figure 1). It implicates that either
a) the system changes from percolation-like behaviour to localised behaviour at a slightly higher η than the phase transition from many small defects to one large cluster or
b) both transitions happens at the same η_t, , but with $2 < \eta_t < 3$.

In the localisation regime, we can measure the roughness w of the fracture surfaces. For reasonably large L, this roughness approaches a simple power law: $w = L^H$.



(a) (b)

FIGURE 6. (a) w as a function of L_x (ln-ln plot). $\eta = 4$, $L_y = 40$. (b) w as a function of L_y (ln-ln plot) for a few combinations of L_x and η.

As can be seen in figure 6, the significant system length scale for w is L_x, the system length perpendicular to the applied voltage. As long as L_y, the system length parallel to the applied voltage, is sufficiently larger than l_y, the roughness is constant over L_y (to the precision of this analysis).

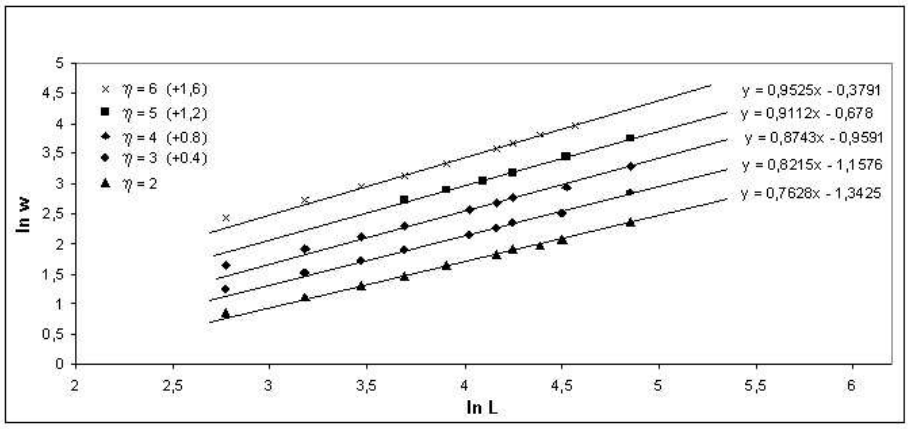


FIGURE 7. w as a function of L (ln-ln plot). The equations for the straight lines best fitted to the data are shown in the figure. The roughness coefficient H corresponds to the gradient of the lines.

Figure 7 shows w as a function of L for $\eta = 2, 3, 4, 5$ and 6. The results for $\eta = 2$ should not be assigned too much weight, because figure 5 (b), as mentioned, indicate that the system is not yet fully localised at $\eta = 2$. We find that H increases with increasing η, as shown in figure 8. More data need to be collected before the exact dependency $H(\eta)$ can be determined. However, the results indicate asymptotic behaviour towards a limit $H \to c_H$ as $\eta \to \infty$, with $c_H = constant$. c_H seems to be close to 1.

As a summary, we can conclude from our data that neither ν nor H are universal quantities for the annealed fuse model. We find that both ν and H increases for

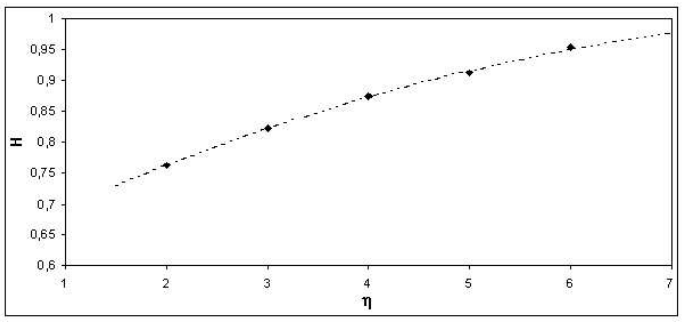


FIGURE 8. H as a function of η.

increasing η. This differs from the results found for, for example, the 'quenched' fuse model, where ν and H are universal[22, 23], and connected by the relation $H = \frac{2\nu}{1+2\nu}$ [19]. We have not yet generated sufficiently amounts of data to determine the relations $\nu(\eta)$ and $H(\eta)$ in detail. But we suggest, based on our data, an asymptotic value $H \rightarrow \approx 1$ as $\eta \rightarrow \infty$ for the annealed fuse model.

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